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"Flow of Single and Two Phase Fluids in Lines"

by

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Introduction

Many physical systems are networks composed of lines connecting
lumped elements. These lines are one dimensional distributed systems
and may be pipes containing fluid, electrical transmission lines, elastic
structures or conductors of heat. Substantial progress has been made in
work on this grant in obtaining greatly simplified mathematical models for
these networks. One key to this approach is the association of a state
vector with each cross section of a line. Suppose that the physical system
is not accelerating. In the case of liquid flow the head and flow rate at
the section are the elements of the vector. For electrical lines the vector
elements are current and voltage. In the case of an elastic line these
elements are displacement and force. Similar vectors are associated with
heat flow.

General Theory

The entire network is described by well-known classical partial
differential equations. Let V_1 and V_2 denote the Laplace transforms of
state vectors associated with two cross sections of lines in the network.
We then have $V_2 = M_{12}V_1$ for a transfer matrix M_{12}, where the elements of M_{12}
depend on the constants of the system. In the case where resistance is
neglected, as is often justified in first approximation studies, the vectors
V_1 and V_2 associated with two cross sections of the same line in which liquid
is flowing are related by a transfer matrix M_{12} whose elements are hyperbolic
functions. If V_1 and V_2 are sections of lines connected by a lumped element,
where this element is a pump, turbine, or restriction such as an orifice, the
matrix M_{12} has only constant elements, whereas if this element is a tank the

elements of M_{12} are rational functions of the Laplace variable s. If terminal conditions are included the transfer function relating any two system variables, considered as system input and system output, is a quotient Q of linear combinations of hyperbolic sines and cosines, where the coefficients are polynomials in s.

For typical system inputs such as step changes, the Laplace transform $F(s)$ of the system output is an expression of the form Q. To obtain the system output it is then necessary to obtain the inverse Laplace transform $L^{-1}(F)$ of $F(s)$.

One task of the project is the derivation of transfer matrices for as large a class of networks as possible.

A second task is concerned with obtaining $f(t)$ where $f(t) = L^{-1}(F)$. Let $F(s) = N(s)/D(s)$ where $N(s)$ and $D(s)$ are the numerator and denominator of $F(s)$ respectively. The denominator $D(s)$ is factored into an infinite product of factors linear in s, i.e. into $K(s+a_1)(s+a_2)\ldots$ for a constant K and zeros $-a_1$, $-a_2$,... of $D(s)$. The problem of factoring reduces to the solution of $D(s) = 0$ for the zeros of $D(s)$. To assist the practising engineer the zeros of $D(s)$ are being determined with the aid of a digital computer for as large a class of functions $D(s)$ as possible. These have been found for functions $D(s)$ of the form $L_1(s) \cosh Ts + L_2(s) \sinh Ts$ where $L_1(s)$ and $L_2(s)$ are linear functions of s and T is a constant. Such expressions arise when a line terminates in an orifice or a tank with a discharge orifice, and thus applies to a wide class of commonly occurring systems.

A third task is to obtain the inverse Laplace transform $L^{-1}(F)$, where the denominator $D(s)$ of F is factored. It is shown that this reduces to the evaluation of integrals of simple combinations of elementary functions. Many of these integrals have been evaluated in closed form.

If the physical network is accelerated the state vectors have an additional element which arises due to the "forcing" nature of the acceleration.

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Example

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The new theory described above is being applied to the problem of POGO oscillations of a missile, such as Saturn V. Studies so far have been restricted to a missile with a single rocket motor, and attention has been focused on one fluid system only, e.g. a LOX tank with helium under pressure and with liquid oxygen, a pipe from this tank to a centrifugal pump, which in turn discharges the LOX into the rocket motor. A complete set of equations has been obtained for the missile in this case. The missile is a network of fluid and elastic elements. Variations in thrust at the rocket motor cause elastic oscillations in the structure of the missile which result in volume changes in the tank as well as vertical displacements of this tank. These volume changes result in pressure changes in the tank which in turn are transmitted through the fluid system to the rocket motor, completing the circuit. The problem has been divided into parts, with final results obtained by superposition. For the transmission of tank pressure changes to the rocket motor, the elastic phenomena in the missile structure are neglected. In treating the missile structure the flexural and longitudinal oscillations of the missile are uncoupled and results subsequently superimposed.

Future Work

Future efforts will be devoted to the following:

1. Derivation of transfer matrices for larger classes of lumped elements.

2. Solution of transcendental equations for more complicated systems. Only simple systems not involving junctions of lines have been treated so far.

3. Evaluation of integrals for inverse Laplace transforms for wider classes of forcing functions (system inputs).

4. Solution of the POGO equations for the simple missile system (single tank, pump, motor) under study.

5. Construction of the POGO equations for the general missile case, and the solution of these equations.

6. Numerical studies based on information requested from NASA for Saturn V.

Objective

The object of this investigation is to obtain as simple rational transfer functions as possible relating system inputs and outputs, where these functions are obtained by lumping distributed systems by mathematical operations on the system partial differential equations, rather than by-passing the partial differential equations by lumping immediately on the basis of physical considerations, such as length. A better understanding of the role of system constants in system performance should result.